Multidimensional Modeling of Radiative Heat Transfer in Scattering Media

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Introduction

There exist various numerical techniques for computing radiative transfer in combustion systems. Among others are included the Hottel zone, Monte Carlo, and flux (or differential approximation) methods. Lockwood and Shah (1981) developed another method, called the discrete transfer method, as an alternative to the well-established techniques. This method not only keeps features of the previous zone, Monte Carlo, and flux methods, but also offers other peculiar advantages such as economy of computation, ease of application to complex geometries, and simplicity of concepts. It is due to these advantages and the easy treatment by numerical techniques used to solve the conservation equations for turbulent flows that this discrete transfer method has been employed, together with solutions of the flow equations, to address such a variety of problems as computing the performance of an industrial glass furnace (Carvalho et al., 1987, 1988), calculating the working conditions of a gas turbine combustor (Carvalho and Coelho, 1989), and predicting the performance of a pulverized fuel fired furnace (Fiveland and Wessel, 1986). The above applications, among others, to full-scale industrial furnaces were very successful. However, the results for a one-dimensional scattering medium did not show the same level of agreement with the benchmark results as did the nonscattering medium predictions (Shah, 1979). Although it is claimed that the method is capable of accounting for scattering in the medium with accuracy, no results have been reported or compared against other benchmark results in multidimensional enclosures (Viskanta and Menguç, 1987).

In the present paper, the discrete transfer model is applied to solve the radiative heat transfer problem in two- and three-dimensional rectangular enclosures containing absorbing-emitting and scattering medium. Results obtained with this technique are compared with other well-established methods, namely the Hottel zone method, the S_n discrete ordinates method, and the P-N differential approximation method. Geometries and surface and gas properties were used in a wide variety of situations to understand the performances of the method better. Required computer times and number of iterations are also reported as a function of number of rays, size of the grid, wall emissivity, and gas scattering coefficient.

Description of the Model

General Features. The fundamental equation for the transfer of thermal radiation may be expressed as:

$$\frac{dI}{ds} = -(k_a+k_s)I + k_a\frac{E_g}{\pi} + \frac{k_s}{4\pi}\int_{4\pi} P(\bar{\Omega}, \bar{\Omega}')I(\bar{\Omega}')d\Omega' \tag{1}$$

where I is the radiant intensity in the direction of $\bar{\Omega}$, s is distance in the $\bar{\Omega}$ direction, E_g is the black body emission power of the gas at temperature T_g, k_a and k_s are the gas absorption and scattering coefficients, and $P(\bar{\Omega}, \bar{\Omega}')$ is the probability that incident radiation in the direction $\bar{\Omega}'$ will be scattered into the increment of solid angle $d\Omega$ about $\bar{\Omega}$. If, for conciseness, we define an extinction coefficient $k_c \equiv k_a + k_s$, an elemental optical depth $ds^* = k_c ds$, and a modified emissive power

$$E^* \equiv 1/k_c\left(k_a E_g + (k_s/4)\int_{4\pi} P(\bar{\Omega},\bar{\Omega}')I(\bar{\Omega}')d\bar{\Omega}'\right), \tag{2}$$

the radiation transfer Eq. (1) may be re-expressed as:

$$\frac{dI}{ds^*} = -I + \frac{E^*}{\pi} \tag{3}$$

For a ray traveling through the domain in study, this equation describes the change of the ray's intensity when passing through an absorbing-emitting and scattering medium.

The discrete transfer method is based on solving Eq. (3) for representative rays that will travel through the considered domain. The directions of the rays are specified in advance (the values of the polar and azimuthal angles, θ and ϕ, are established) and they are traced along paths between the two boundary walls. The enclosure is subdivided into control volumes or cells. The intensities along each of the chosen directions are solved for, and the values of the intensities entering and leaving each cell are calculated.

Consideration of In-Scattering. The in-scattering term, like the emissive power (see Lockwood and Shah, 1981), is presumed constant over each small control volume. The in-scattering energy contributed by each ray that crosses the control volume, through which the ray being traced is "traveling," is discretized as (see Lockwood and Shah, 1981):

$$I_s \approx \frac{k_s}{\pi} P(\bar{\Omega}_{P_I Q_I}, \bar{\Omega}'_{P_I Q_m})\, I(\bar{\Omega}'_{P_I Q_m})_{\text{avg}}\, \delta\Omega'_{P_I Q_m} \tag{4}$$

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where $I(\bar{\Omega}'_{P_f Q_m})$avg is a value averaged over the control volume; the arithmetic mean of its values at k and $k+1$ (entering and leaving the control volume in the Ω' direction) would be convenient. The in-scattering term will then be given by the summation of the contributions of all the rays that crossed the control volume. This means that the memorized averaged energies of all rays that crossed each cell in the previous iteration will now be used to evaluate the in-scattering term. It is then to be expected that the CPU time necessary for each iteration will increase and the number of iterations may also increase (convergence of the in-scattering terms will additionally be required).

Results

In this section we consider various illustrative examples for two- and three-dimensional rectangular enclosures.

Two-Dimensional Rectangular Enclosures. For this type of geometry, two cases are examined: (*i*) scattering in a black enclosure, (*ii*) scattering in a gray enclosure. These examples were chosen to benchmark the discrete transfer method against the S_n discrete-ordinate method, the *P-N* differential approximation, and the Hottel zone method. The results obtained with the discrete transfer method were compared with the S_2, S_4, and S_6 approximation results obtained by Fiveland (1984), with the results obtained by Ratzel and Howell (1983) using a P_3 approximation and with the results obtained by Larsen (1981) using the Hottel zone method.

Results are presented using nondimensional values; radiant intensities are normalized using a characteristic emissive power, while coordinate directions are normalized with a characteristic length.

(*i*) *Pure Scattering in Black Enclosures.* The studies focused on radiative transfer with isotropic scattering in a square enclosure with black walls and a scattering cross section of unity. The emissive power of surface 1 is unity, while the emissive powers of surfaces 2, 3, and 4 are zero. This geometry was analyzed by Larsen (1981), Ratzel and Howell (1983), and Fiveland (1984).

For these cases, the discrete transfer method using typical values of size mesh and number of rays for two-dimensional problems without scattering led to unsatisfactory results. As suggested by Viskanta and Menguç (1987), this behavior of the discrete transfer method may result from the so-called "ray effect" (Lathrop, 1968, 1971). In scattering media each ray is not only responsible for carrying information from one wall to another, but also for contribution to the in-scattering term in Eq. (3). With this in mind it is understandable that in scattering media the ray effect tends to be more notorious. To overcome this problem, an increase in the size of the mesh and especially in the number of rays used was necessary to obtain the results presented in Fig. 1. The number of rays and the size of the mesh are intimately linked and optimization has to be achieved in order to get an acceptable number of rays crossing each cell. The number of rays crossing each cell should be maximized without loosing accuracy due to lowering the number of grid cells. Independent studies of the grid and number of rays should be performed in conjunction.

Figure 1 shows the comparison of centerline distributions of radiant intensities for different rectangular enclosures. Discrete transfer predictions, using a 10×10 mesh and 64 rays per node, are compared with the P_3 solution method results and the S_4 discrete ordinates solutions. The results presented by Modest (1975) using the zone method are also shown. For high aspect ratios (hot wall much larger than side walls) an increase in the number of ϕ's was used leaving the number of θ's similar to the values used in typical two-dimensional non-scattering problems so that rays (coming from the hot wall) could reach the side zones of the domain. For $\alpha = 0.1$ the opposite was done.

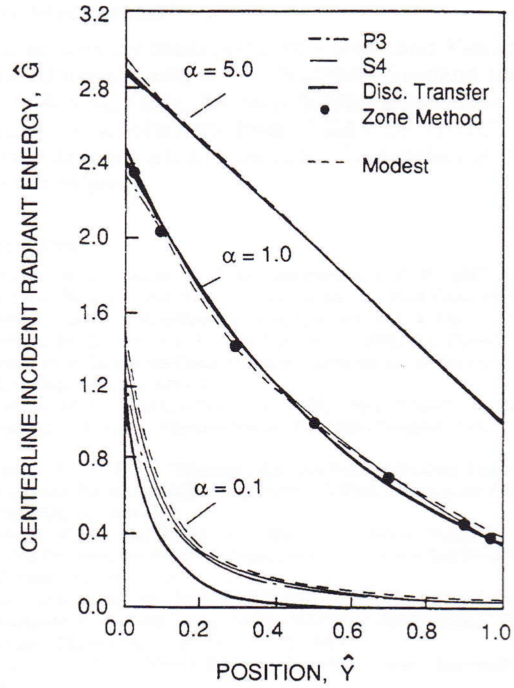

Fig. 1 Centerline incident radiant energy for different aspect ratios in an square enclosure with a scattering medium; surface 1; $\epsilon = 1$, $E_w = 1$; surfaces 2-4: $\epsilon = 1$, $E_w = 0$; $k_s = 1$; $k_a = 0$

The discrete transfer method presents very good results both for high and intermediate aspect ratios. For very low aspects ratios ($\alpha = 0.1$: emitting wall is ten times smaller than side walls), as we move away from the hot wall, fewer rays (carrying information about the emitting wall) will reach the cells. For this reason, the incident radiation energy values are underpredicted by the discrete transfer method when compared with the other ones.

(*ii*) *Pure Scattering in a Gray Enclosure.* Predictions of mean radiant intensity were obtained using the discrete transfer method for a radiative gray square enclosure with isotropic scattering. A scattering cross section of unity was assumed. The predictions were compared with the discrete-ordinate S_n solutions (Fiveland, 1984), Hottel's zone method (Larsen, 1981), and the P_3 differential results (Ratzel and Howell, 1983).

As in the previous case, a 10×10 mesh and a large number of rays had to be used (64 rays power wall node). From Fig. 2(*a*) it can be noticed that the results obtained with the discrete transfer method are satisfactory, being very similar to the ones obtained by the S_6 discrete ordinates predictions.

In Fig. 2(*b*) heat transfer rates to the hot surface are shown for different wall emissivities. The discrete transfer predictions clearly follow the S_4 and S_6 solutions (which, according to Fiveland, 1984, are the solutions that adequately predict the surface heat transfer rates) for both gray and black enclosures.

Three-Dimensional Rectangular Enclosures. The three-dimensional absorbing-emitting and isotropic scattering case studied is based on the idealized furnace presented by Menguç and Viskanta (1985).

The results obtained with the discrete transfer method, using a $10 \times 5 \times 5$ mesh and 64 rays per wall node, were compared with the S_2, S_4, S_6, and S_8 discrete ordinates predictions (Jamaluddin and Smith, 1988) and the zone model results (Truelove, 1987). Increasing the number of rays or the size of the mesh did not change the results more than 1 percent.

Figure 3 shows the heat fluxes to the firing end and exit end

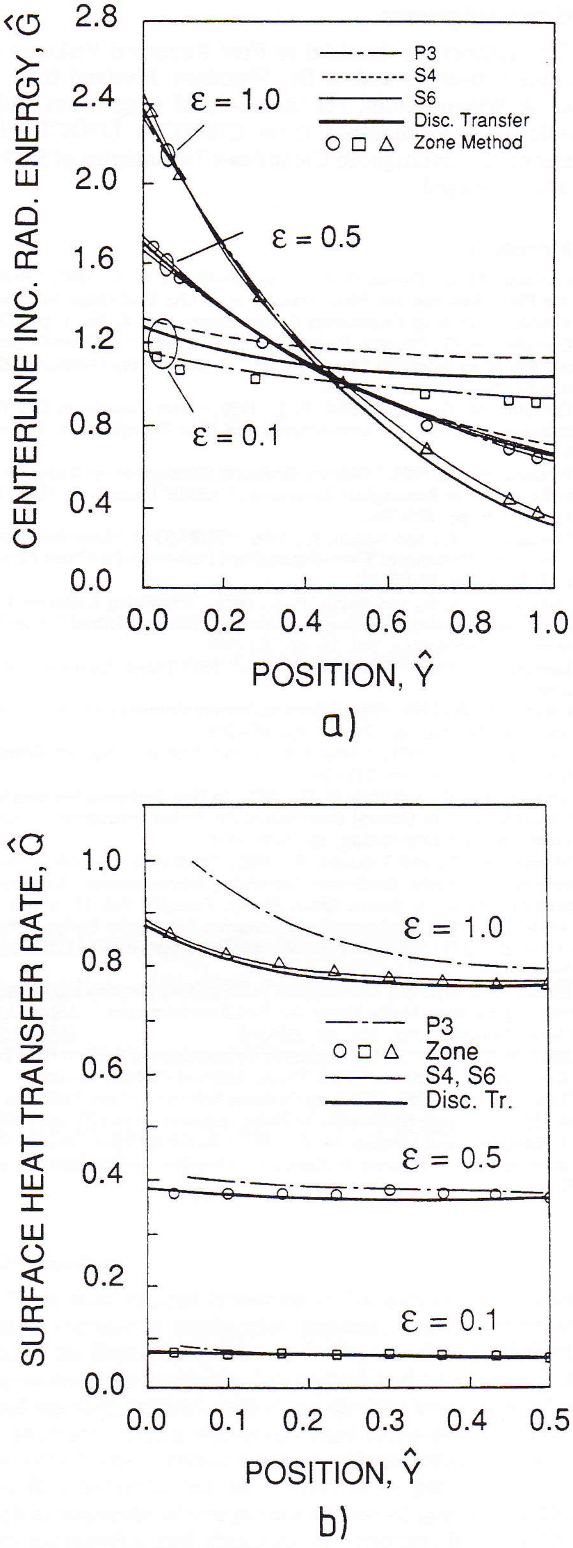

Fig. 2 Square gray enclosure with isotropic scattering medium: (*a*) centerline incident radiant energy; (*b*) hot surface heat transfer rate; surface 1: $E_w = 1$; sufaces 2-4: $E_w = 0$; $k_s = 1$; $k_a = 0$

walls of the furnace. The results clearly show the good performance of the discrete transfer method for the calculation of heat transfer in a three-dimensional enclosure containing as absorbing emitting and isotropic scattering medium.

The results obtained with the S_4, S_6, and S_8 discrete-ordinate method are very similar to the discrete transfer predictions, and due to the increase in computational effort, as reported by Jamaluddin and Smith (1988), no advantages are obtained by using S_6 or S_8 when compared with the S_4 method.

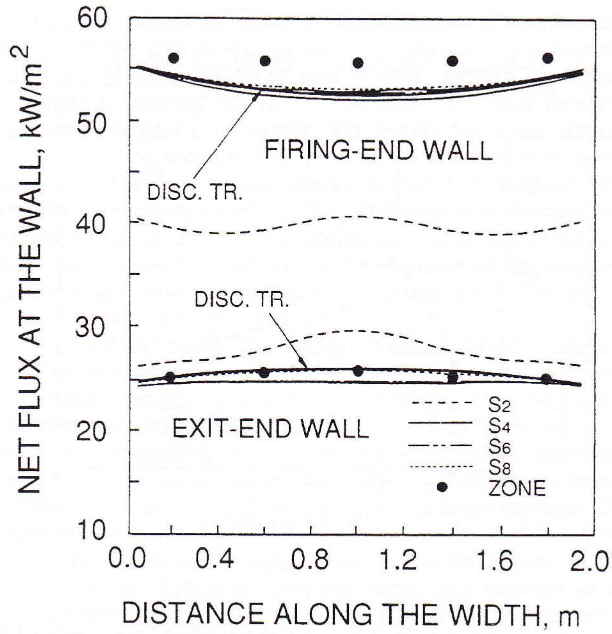

Fig. 3 Predicted radiative heat fluxes at the firing end and exit end walls of the rectangular enclosure idealized by Menguç and Viskanta (1985)

Required Computer Times

To evaluate the required computer times and number of iterations for different situations, the test case considered was the same as previously presented for two-dimensional enclosures containing an isotropic scattering medium.

In Fig. 4(*a*) the CPU time required in a VAX 6000 machine to obtain a preset error (heat balance error at the wall surface not exceed 0.1 percent) are shown for different computational grids (64 rays per wall node were used) and different number of rays (a 10×10 grid was used). Whenever better accuracy is desired, it is preferable to increase the number of rays used, rather than the size of the grid. This is due to the fact that an increase in the size of grid implies an increase in the number of walls cells and therefore also in the number of rays emitted.

In Fig. 4(*b*), the iterations needed to obtain the same present error mentioned above are shown for different wall emissivities (pure isotropic scattering was considered: $k_s = 1.0\ \mathrm{m}^{-1}$) and different scattering coefficients (black walls were assumed). A 10×10 uniform grid and 64 rays per wall node were considered. As expected, the number of iterations required to obtain the solution increases with the value of the scattering coefficient. Wall emissivities also influence the number of iteractions required for convergence. For high values of the wall emissivity, the number of iteractions required is low; nevertheless, as the emissivity approaches zero the number of iteractions increases considerably.

For comparisons with other radiative methods, namely the discrete ordinates method, Fiveland (1984) presents, for the same test case, the CPU times required to obtain the S_2, S_4, and S_6 solutions for different wall emissivities. The CPU times for the discrete transfer and S_4 methods are of the same order of magnitude.

Conclusions

The discrete transfer solutions were compared to other solution methods for two- and three-dimensional rectangular

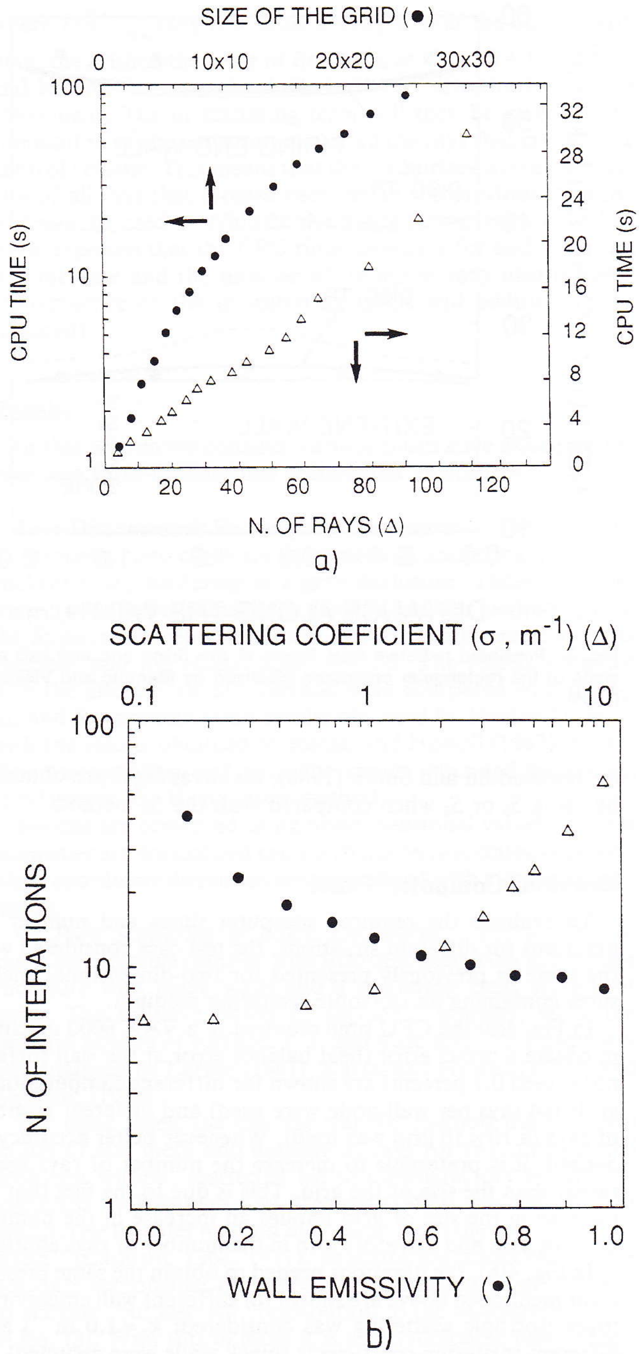

Fig. 4 Computational load: (*a*) CPU time required to achieve convergence for different computational grids and number of rays; (*b*) iterations required to achieve convergence for different wall emissivities and scattering coefficients

enclosures containing an absorbing-emitting and isotropic scattering medium.

The discrete transfer method has shown very good results when scattering was considered. However, the number of representative directions used had to be increased, when compared to normal situations where scattering is not considered. Only for very small aspect ratio (where the emitting wall is much smaller than the side walls) did the discrete transfer method present significant deviations. This behavior of the discrete transfer method results from the "ray effect." It was found that, for the studied cases, the best remedy to reduce the ray effect is to increase the number of rays that cross each cell. In order to avoid a prohibitive CPU time, an optimization of the number of the grid cells and rays should be sought.

Acknowledgments

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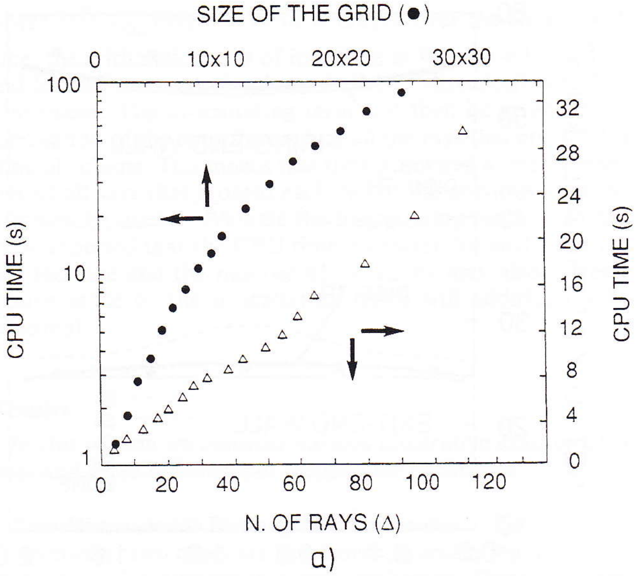

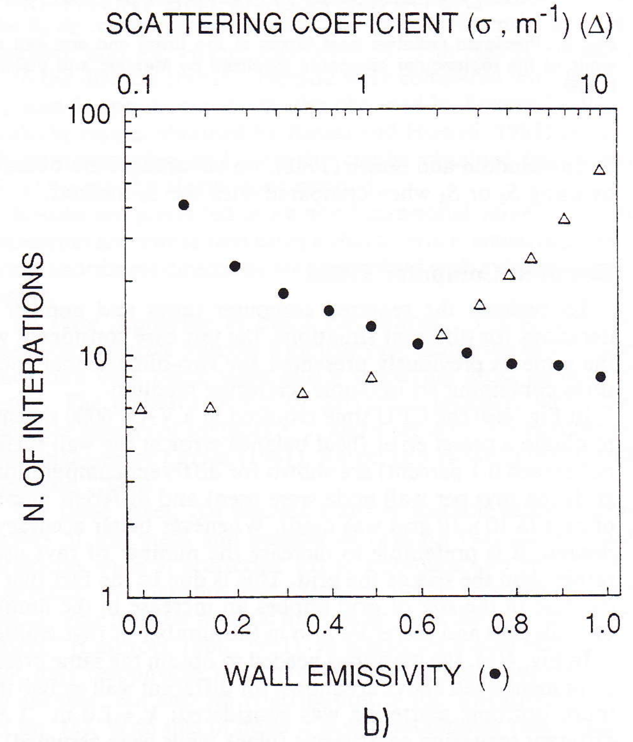

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